SOME EXPERIENCES WITH ALGORITHMS IN MUSICAL COMPOSITION

Gordon Monro[1]

Abstract

This paper discusses the author's experience in using algorithms in musical composition, both for determining notes and for creating timbres. The paper concludes with some comments on the relationship between music and mathematics.

1. INTRODUCTION

Algorithms in musical composition can be divided into two main classes:

1. Algorithms that determine the notes that make up the piece.
2. Algorithms that produce the actual waveforms or sounds that are heard.

This paper discusses my experience with algorithms in both of these classes, and ends with some comments on the relationship between mathematics and music.

2. NOTE-BASED ALGORITHMS

The term *algorithmic composition* is usually taken to mean the use of algorithms that generate the notes of a composition. The output could be a printed score, or a MIDI file that can be played back on a synthesiser. The use of computers in this connection goes back to the "Iliac Suite" [9] of 1956, and there is a considerable earlier history of mechanical devices and pencil-and-paper procedures. (See [13] and [18] ch. 18 for general discussions of algorithmic composition.)

One can divide the uses of note-based algorithms into two categories: attempts to mechanise (capture, model) an existing musical style, and attempts to create new styles, to explore new compositional territory.

A great deal of work has been done on mechanising existing styles. Some of this work has been done for musicological reasons, aiming to understand a particular style (e.g. David Cope's EMI program [5], which he used to create a new Mozart symphony [10]). Other work in this area has been done for commercial reasons, for example to develop products that allow untrained people to create accompaniments in a given style (as with Band-in-a-box [17]).

My own interest has been in using algorithms to explore new compositional territory. Again, there are two ways in which algorithms can be used to this end. One can use

[1] School of Mathematics and Statistics, University of Sydney F07, NSW 2006.
Email: (see website below)
Web: http://www.gordonmonro.com/

an algorithm to generate fragments, which the composer takes as source material and proceeds to combine and process "intuitively". Or one can use an algorithm to generate the whole piece "at the press of a button".

I have worked in both of these ways. In the piece "Study in Dimension 1.245" (1992) I used the method of *fractal interpolation* as described by Barnsley ([2] ch. 6), using his deterministic algorithm ([2], program 3.8.1). Given a piecewise linear function, this method constructs a fractal by an iterative process, the output at each stage being another piecewise linear function. By taking the x axis as time and the y axis as pitch (quantised to the nearest tempered semitone), the process produced a sequence of increasingly elaborated melodies, all with the same basic structure. I then proceeded to "filter" these melodies (deleting certain sets of pitches, using in fact scales obtained from a Hadamard code) and then manipulated the results intuitively to make the piece.

I have constructed two pieces using the following piece of "recreational mathematics" (due, I believe, to A.J. van der Poorten) as a base. The formula

$$\left(\sum_{k=0}^{n} e^{2\pi i k^3/1086}\right)_{n=0,\dots,1086}$$

describes a sequence of complex numbers, which we regard as points in the plane. Connecting consecutive points in the sequence with line segments gives the diagram in Figure 1 below. (The number 1086 was chosen because it gave an interesting pattern.)

I used the sequence of y values as pitches in a piano piece "Prelude and Toccata on the Number 1086" (1996) (the x values controlled other aspects of the piece). The result was largely algorithmically generated, but I had to thin out a lot of the chords by hand before it was playable. I have to say that the piece was not very successful: I think the basic idea was probably not strong enough.

I used the same diagram to make a tape piece "Mandala 1086" (1996). Here the y values were used to determine untempered pitches, with no relation to the usual scale, and each note was a sine wave about 30 seconds long. At any one time there were maybe 80 of these sine waves contributing to the sound. This piece was generated entirely "at the press of a button".

Another piece generated entirely automatically was "Peer Pressure" (2000), which used an algorithm modelling a process which apparently provides a mechanism explaining the behaviour of some types of firefly, where large groups of insects all flash in unison [3].

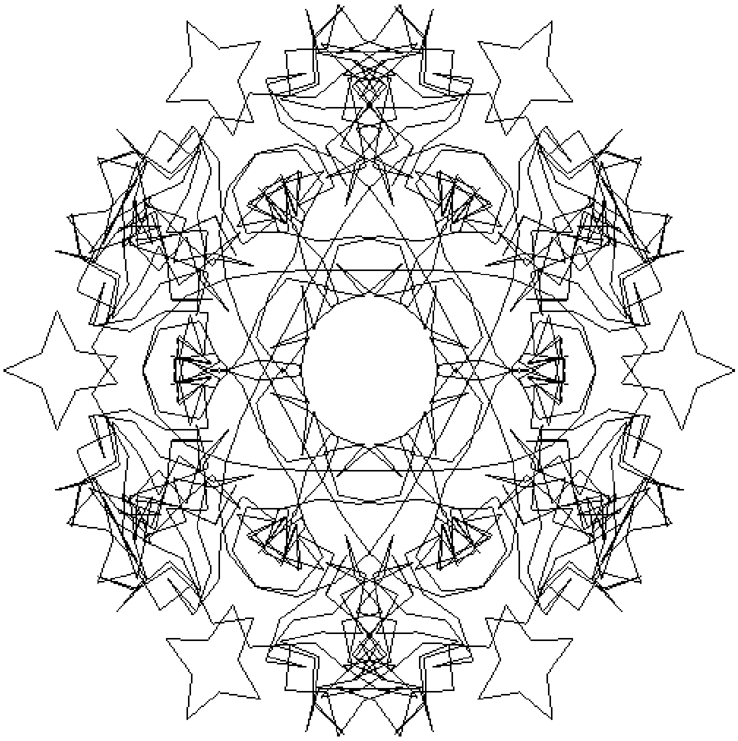

Figure 1.

3. ALGORITHMS FOR CREATING WAVEFORMS OR SOUNDS

Algorithms for creating waveforms come under the heading of *digital sound synthesis.* Two motivations for this work are: to reproduce existing acoustic instrumental timbres; to create entirely new sounds. These two motivations are related, because one of the challenges in producing new sounds is to make them as interesting as real instrumental timbres.

Cutting across this classification by motivation is one of technique. The following classification is not entirely logical, but is approximately ordered by historical order and by increasing computer power required.

1. "Mathematical" algorithms.
2. Algorithms aimed at matching perceptual features.
3. Digital signal processing algorithms.
4. Physical modelling algorithms.

"Mathematical" algorithms

By "mathematical" algorithms I mean algorithms based on a mathematical formula which is easy to compute, and has a small number of adjustable parameters which lead to the production of a wide variety of sounds. The best-known example is FM synthesis ([18] pp. 224–250), using a formula like

$$\text{output} = A\sin\left(2\pi\left(f + b\sin(2\pi\, g t)\right)t\right),$$

where f is the *carrier frequency* and g the *modulation frequency*. The Yamaha FM synthesisers such as the DX7 realised extensions of this formula allowing, for example, further modulations.

“Perceptual” algorithms
With these algorithms, the aim was to match perceptual features of a natural sound, without necessarily reproducing the exact waveform of the sound. The best-known example is the *fonction d’onde formantique* (FOF) synthesis ([18] pp. 296–306), developed to synthesise the singing voice. The boundary between these algorithms and the “mathematical” algorithms is vague, as, for example, considerable work was done on trying to imitate acoustic instruments with FM synthesis.

Digital signal processing (DSP) algorithms
These don’t quite belong in this sequence, as these algorithms are more aimed at processing a recorded sound than creating a new one from scratch. However, cheaper computer memory allowed the development of samplers that could record and reproduce natural sounds. For a time this made the synthesis of imitations of natural sounds rather pointless, and modifications of natural sounds became, and remain, very popular.

The most powerful DSP methods are the *analysis-synthesis methods*. The *phase vocoder* ([18] pp. 566–570) cuts up the sound into short sections and applies a discrete Fourier transform to each section. The resulting Fourier information can be modified and used to synthesise an altered version of the sound. An extension is the *tracking phase vocoder* or *McAulay-Quatieri technique* ([18] pp. 570–573) which uses the Fourier information to track prominent sinusoidal partials in a sound. Again, the information describing these partials can be modified and used to synthesise an altered version of the sound.

Physical modelling algorithms
These algorithms are a return to the synthesis of sound *ab initio* (rather than processing a recorded sound), and are based on mathematical models of string, wind and percussion instruments or the human voice ([18], 265–292). They require a great deal of computing power, and have only appeared in commercial synthesisers relatively recently [22].

4. Applications of digital sound synthesis in composition

The relevance to musical composition of algorithms creating sounds is that over the last 200 years or so, timbre has become an increasingly important element in musical composition, to the point where in many works it is at least an equal partner with pitch and duration.

In my own work I have explored timbres by returning to “mathematical” algorithms: taking a mathematical formula or idea and using it to generate sounds in a spirit of curiosity, without attempting to imitate natural sounds. The availability of increased computing power means that much more complex algorithms can now be explored than was the case in 1973, when FM sound synthesis was introduced.

I have used the fractal interpolation procedure mentioned above to create waveforms rather than melodies [14]. I was able to get a variety of timbres, but the best results came when I used extremely low frequencies. The piece “Dry Rivers” (1995) [16] used frequencies ranging from 1/6 Hz to 14 Hz. These would normally be inaudible, but the waveforms were so complex that they emerged as percussive sounds. One of the waveforms used is shown in Figure 2.

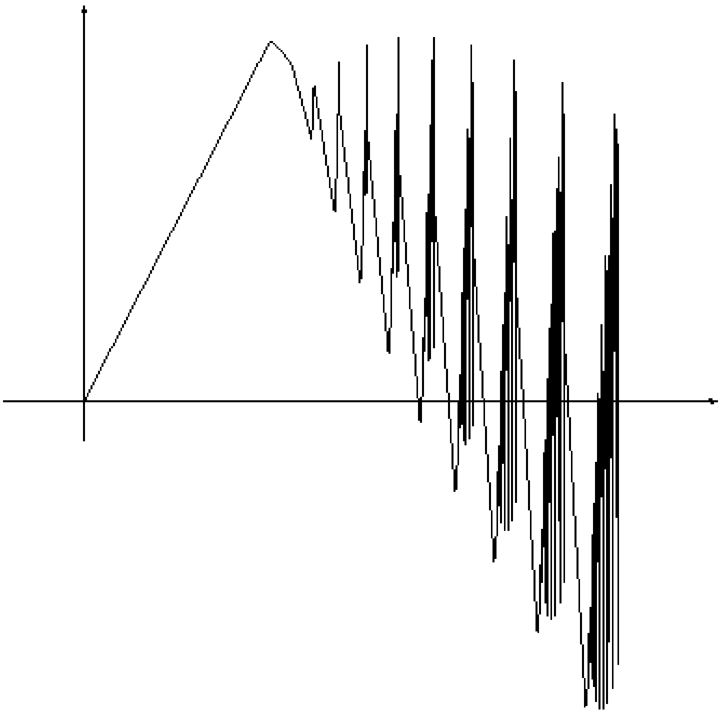

Figure 2.

In “Dry Rivers” I was trying to get away from “notes”: the individual cycles of the waveforms arguably function as notes, and otherwise there are “phrases” varying in length from 34 seconds to the length of the whole piece (12 minutes).

Another "mathematical" technique I have employed is the use of chaotic oscillators. My starting point was a simplified model of the sun's magnetic field [4], but I took a very primitive discrete version of the equations, and the final results owes much more to the discretisation than the original continuous equations. However, a great variety of timbres resulted: one of the waveforms is shown in Figure 3.

The piece "The Voice of the Phoenix (1)" (1996) [15] consists entirely of the output of 13 oscillators, with randomly varying parameters. Since the oscillators are unpredictable, they sometimes ran away, and my program contained routines to shut them down if this happened, and restart them when the parameters got back to values permitting stable oscillation. Thus the individual oscillators do not all sound all the time. Apart from this, there are no "notes" in the piece, and the whole thing was generated automatically.

The piece "The Voice of the Phoenix (2)" (1997), for bass flute and tape, used the same oscillators (this time 17 of them), but I did modify the output by hand, mostly to alter the relative volumes of the oscillators. The flute part was composed after the tape part, and to some extent attempts to imitate its timbres.

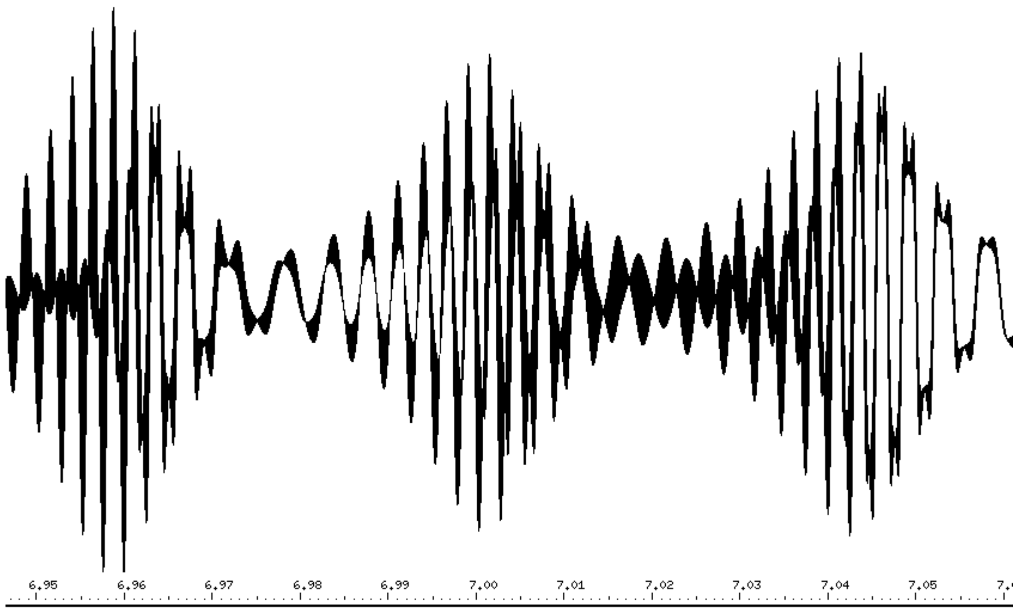

Figure 3.

I have made a foray into DSP by experimenting with wavelet transforms as an analysis-synthesis technique. So far the results have been a bit limited, but I did use the results in "Lament over Jerusalem" (1998), a piece for cello and tape, where the tape part consists entirely of modified cello sounds. The modifications were fairly simple-minded, consisting of such things as inserting a zero after each number in the wavelet transform and then resynthesising. There has been some use of wavelets in sound compression and audio noise reduction, but there seems to have been very little use for sound transformation apart from a little burst of activity around 1990 [12].

5. Mathematics and music

Does work with algorithms in composition shed any light on the relationship between mathematics and music?

The idea of this relationship of course goes back to antiquity, and its nature is an old problem. "Mathematics and music are really the same thing" seems to be a sentiment more popular among lay people than either musicians or mathematicians.

Xenakis ([21] pp. 99–101) has given a table of "correspondences between certain developments in music and mathematics". He sees the prototype of logarithms in the relationship between string lengths and musical interval (the theory dates from 300 BC), the musical staff (1000 AD) as the forerunner of a two-dimensional coordinate system, and the fugue as "an abstract automaton used two centuries before the birth of the science of automata". More recently the influence has run the other way: Xenakis gives a list of various applications of mathematical structures and ideas in musical composition since 1950, many due to Xenakis himself.

Xenakis also foresees the possibility that "the arts would consciously 'set' problems which mathematicians would then be obliged to solve by the invention of new theories." ([21] p.3).

On the other hand Rothstein near the start of his book on music and mathematics ([19] p.5) states "Connections between the two [mathematics and music] have had almost no importance for the development either of math or of music; mostly any relationships have been irrelevant to their practitioners and creators, and mystically vague to everyone else."

My own tentative view is as follows.

The commonalities between mathematics and music appear to me to be firstly abstraction and pattern, and secondly intellectual curiosity, observing the working-out of some process. However, there are evidently very substantial elements present in music but not mathematics, and *vice versa*.

In music it is clear that human physiology and psychology play a very important rôle. In fact, music is presumably a secondary phenomenon or by-product of our physical nature. Our auditory system surely evolved to help us identify things in the environment as friends, enemies, things that we could eat, or things that might eat us, rather to give us the abstract pleasure of listening to music.

In composition there is (or was) a rule that a sequence ("the more or less exact repetition of a passage at a higher or lower level of pitch" [11]) should not contain more than three repetitions. This cannot be a purely mathematical rule, but must relate to human psychology. Again, it is relatively easy to produce from fractals images that are pleasing to the eye: my experience indicates that it is much harder to produce from fractals music that is pleasing to the ear. This indicates that the abstract pattern is not enough to make an artwork. Finally, a very large amount of music, including much Western "abstract" music, is related to a corporeal activity: dance.

This connection of music with immediate human experience is why I largely disagree with Babbitt's article "Who cares if you listen" [1]. Babbitt argues that a lay person does not expect to understand an advanced mathematics lecture, so why should a lay person expect to understand an advanced musical composition? However, it appears that essentially no one (not even the composers) can apprehend the sort of total serialist music that Babbitt was considering. Herbert von Karajan ([6] p.82), speaking 25 years after Babbitt's article was written, stated that he has fairly often had the experience that composers were amazed at how their scores sounded, and further that there were fewer than five people in the world (and he did not consider himself one of them) who could determine what a piece of modern music would sound like from reading the score.

On the other side, I see two defining characteristics of mathematics not present in music: *universal significance* and *inevitability*.

Hardy [8], in considering why mathematics is more significant than chess, saw the significance of a mathematical idea in its connections with other parts of mathematics. This by itself would not prevent mathematics from just being a very large game, though it is true that it is remarkably interconnected. However, many mathematical ideas have deep roots in the physical sciences. Wigner [20] and Hamming [7] have considered the "unreasonable effectiveness" of mathematics in the natural sciences. Neither author can explain this effectiveness, but there is no doubt that it exists. I do not see any such cosmic universality in music, which often enough is bound to one (human) culture or sub-culture.

Related to the universal significance of mathematics is what I think of as its inevitability: it is true that sporadic simple groups exist, and no amount of wishful thinking on my part will change this result, even if I object to sporadic objects on principle. At least in my experience, music does not have this sort of inevitability: there is room for arbitrary decisions. When I heard a respected Australian composer (whose undergraduate degree included a major in pure mathematics) talking about "the 15 mathematical processes" in the first section of one of his pieces, my immediate thought was "this is not mathematics". That was not intended as a criticism, but it is an indicator of a profound difference between mathematics and music. I cannot imagine what *proof* would mean in music.

Form my point of view, perhaps the greatest mystery about mathematics is why elegance, or beauty, should be a guide to truth.

6. Conclusion

Rothstein [19] concludes essentially that the similarities of mathematics and music are meta-similarities. "This life of the mind expands outward to the world of nature and inward to the act of creation. It encompasses a linked set of worlds in which metaphors are resonantly connected. Mathematics and music are products of that life…" ([19] p.242). And (after a discussion of what "complete" and "consistent" might mean when applied to music), "Musical truth involves something about the ways a composition is complete, consistent, and open to mappings in our various worlds of sense and thought" ([19] p.226).

Xenakis ([21] p.6) would go further and see art, combining inference, experiment and "revelation", as a universal guide to the sciences.

I have to say that I find it much easier to agree with Rothstein, and see the arts as proceeding in parallel with the sciences, with some common concerns but with many differences, than to see one as a guide for the other. Nonetheless, the rôle of beauty in mathematics remains to be explained.

At a much less abstract level, algorithmic composition (at both the note level and the waveform level) provides specific mappings of mathematical processes into musical processes. Where such a mapping is musically successful, we have arguably a closer connection than a meta-similarity: we have a pattern that works in some way in both mathematics and music, even if the mode of apprehending the pattern is quite different in the two cases. Nevertheless, the outcome of an algorithmic compositional process is a musical work, not a mathematical paper!

References

[1] M. Babbitt, "Who cares if you listen", *High Fidelity*, 8 no 2, 38–40 and 126–127, (Feb. 1958), reprinted in E. Schwartz and B, Childs (eds) *Contemporary Composers on Contemporary Music*, Holt, Rinehart and Winston, 1967, 243–250, and also in G. Chase (ed) *The American Composer Speaks: a Historical Anthology*, Louisiana State University Press, 1966, 234–244.

[2] M. Barnsley, *Fractals Everywhere*, Academic Press, (1988).

[3] S. Bottani, "Pulse-coupled relaxation oscillators: from biological synchronization to self-organised criticality", *SIAM Journal on Applied Mathematics*, 50 no. 6, 1–4, (1990).

[4] F. Cattaneo, C.A. Jones and N.O. Weiss, "Periodic and aperiodic behaviour in stellar dynamos", *Solar and Magnetic Fields: Origin and Coronal Effects*, ed. J.O. Stenflo, International Astronomical Union, 1983.

[5] D. Cope, *Computers and Musical Style*, A-R Editions, Madison, Wisconsin, 1991.

[6] H. Götze and R. Wille (eds), *Music und Mathematik: Salzburger Musikgespräch 1984*, Springer-Verlag, Berlin, (1985).

[7] R.W. Hamming, "The unreasonable effectiveness of mathematics", *American Mathematical Monthly*, 87 no. 2, 81–90, (1980).

[8] G.H. Hardy, *A Mathematician's Apology*, Cambridge University Press, (1940). Reprinted with a foreword by C.P. Snow, 1967.

[9] L. Hiller and L.M. Isaacson, *Experimental music: composing with an electronic computer*, McGraw-Hill, New York, (1959).

[10] Holmes, Bob, “Requiem for the soul”, *New Scientist*, 9 August 1997, also at http://www.newscientist.com/hsplus/insight/ai/requiem.html (1997).

[11] M. Kennedy (ed.), *The Concise Oxford Dictionary of Music*, 4th edition, Oxford University Press, (1996).

[12] R. Kronland-Martinet, “The wavelet transform for analysis, synthesis and processing of speech and music sounds”, *Computer Music Journal* 12 no 4, 11–20, (1988).

[13] G. Loy “Composing with computers—a survey of some compositional formalisms and music programming languages”, in *Current Directions in Computer Music Research*, ed. M.V. Mathews and J.R. Pierce, MIT Press, 1989.

[14] G. Monro “Fractal Interpolation Waveforms”, *Computer Music Journal* 19 no 1, 88–98, (1995).

[15] G. Monro, “Taming chaotic waveforms”, *Proceedings of the Interface 97 Conference*, School of Music, University of Auckland, 1 – 9, (1997).

[16] G. Monro, “This is art, not science”, *Leonardo Music Journal,* 7, 77–80, (1997).

[17] PG Music, “Band-in-a-box”, http://www.pgmusic.com/bandbox.htm .

[18] C. Roads *et al*, *The Computer Music Tutorial*, MIT Press, Cambridge, Massachusetts, (1996).

[19] E. Rothstein, *Emblems of Mind: The Inner Life of Music and Mathematics*, Avon Books, New York, (1995).

[20] E. Wigner, “The unreasonable effectiveness of mathematics in the natural sciences, *Communications on Pure and Applied Mathematics*, 13 no 1, 1–14, (1960).

[21] I. Xenakis, *Arts/Sciences: Alloys (The Thesis Defense of Iannis Xenakis)*, Pendragon Press, New York, (1985).

[22] Yamaha Corporation, “VL70-m”, http://www.yamahasynth.com/pro/v170m/ .